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The effects of similarity breaking on the intracluster medium

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ABSTRACT
We construct a family of simple analytical models of galaxy clusters at the present epoch and compare its predictions with observational data. We explore two processes that break the self-similarity of galaxy clusters: systematic variation in the dark matter halo concentration and energy injection into the intracluster gas, through their effects on the observed properties of galaxy clusters. Three observed relations between cluster properties and temperature are employed to constrain the model; mass, slope of gas density profile (β) and luminosity. The slope of the mass-temperature relation is found to be reproduced by our model when the observed variation in concentration is included, raising the logarithmic slope from the self-similar prediction of 1.5, to that of the observed relation, ~ 2. Heating of the intracluster gas is observed to have little effect on the mass-temperature relation. The mean trend in the β-temperature relation is reproduced by energy injection in the range 0.5-0.75 keV per particle, while concentration variation is found to have only a small effect on this relation. Excess energies calculated for individual systems from the β-temperature relation suggest that the lowest mass systems may have excess energies that are biased to lower values by selection effects. The observed properties of the luminosity-temperature relation are reproduced by the combined effects of excess energy and a trend in the dark matter concentration. At high masses the observed variation in dark matter concentration results a logarithmic slope of ~ 2.7 compared to recent observations in the range 2.6-2.9, whilst the observed steepening of the relation in galaxy groups is predicted by the model when heating in the range 0.5-0.75 keV per particle is included. Hence a combination of energy injection and dark matter concentration variation appears able to account for the mean trends in all the observed relations. Scatter in the energy injection and concentration may account for a large proportion of the scatter in the observed relations.

Key words: galaxies: clusters: general - intergalactic medium - X-rays: general

1 INTRODUCTION

In the hierarchical clustering model for the formation of structure in the universe, small scale perturbations collapse into virialized objects and then cluster together to form successively larger virialized structures. Simple physical models of gravitational collapse along with shock heating of the gas would suggest that these virialized structures should be approximately scaled versions of one another (Navarro et al. 1995). In the absence of other physical process such as heating or cooling, the gas and dark matter halos will be almost self-similar. It is possible for energy to be transferred between the dark matter and the gas during mergers (Pearce et al. 1994) which is believed to be responsible for the cores observed in cluster gas density profiles. However this is not usually assumed to act in a way which would break self-similarity.

This expected self-similarity has already been observed to be broken in several respects. The surface brightness profiles of galaxy clusters have been observed to flatten in low mass systems (Ponman et al. 1999; Helsdon & Ponman 2000b) and their gas density, entropy and energy have been shown not to be self-similar (Lloyd-Davies et al. 2000). The luminosity-temperature relation for galaxy clusters is also observed deviate from its expected self-similar behaviour (Edge & Stewart 1991; Markevitch 1998; Helsdon & Pon-

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man 2000a). These effects are usually attributed to energy injection into the intracluster medium (ICM) from galaxy winds (Lloyd-Davies et al. 2000). However there is some controversy over whether galaxy winds can provide the energy necessary to heat the ICM (Valageas & Silk 1999; Wu et al. 2000; Bower et al. 2001). An alternative often promulgated is that AGN are responsible for the injected energy. However while the amount of energy available from AGN is very large, it is not entirely clear how they could transfer it into the ICM. AGN have emitted large amounts of energy in the form of electromagnetic radiation, especially in the ultraviolet, but this will not heat gas to the required temperatures. Mechanical energy transfer through AGN jets seems the only possible option at present, but it is not clear how much energy is available in this form or whether it can be transferred efficiently into the ICM. It should be noted that preliminary Chandra results (Fabian 2001) do not suggest that widespread heating of the ICM by AGN is occurring at low redshift.

The amount of energy injection inferred from the observed breaking of similarity is also controversial. Wu et al. (2000) suggest a value of 1.8-3.0 keV per particle is needed to explain the steepening of the luminosity-temperature relation from the self-similar prediction of $\sim T^2$ to the observed value of $\sim T^3$. However Lloyd-Davies et al. (2000) have measured a mean excess energy of 0.44 keV per particle in a sample of 20 galaxy clusters and groups using analytical models fitted to X-ray spectral images. There would also seem to be a theoretical argument against energy injection of much greater than 1 keV per particle since it is difficult to see how galaxy groups with mean temperatures less than 1 keV could have X-ray haloes, whereas a number of such systems are known to exist (Ponman et al. 1996; Helsdon & Ponman 2000b). It therefore seems unlikely that energy injection into the the ICM can be the explanation for the steepness of the *cluster* luminosity-temperature relation or that any energy injection can greatly exceed 1 keV per particle.

Breaking of simple self-similarity in the dark matter constituents of galaxy clusters has also been observed. The dark matter concentration of galaxy clusters systematically decreases with increasing cluster mass (Wu & Xue 2000; Sato et al. 2000; Lloyd-Davies & Ponman 2002). This effect has been observed for some years in numerical simulations (Navarro et al. 1997; Moore et al. 1999). It is generally interpreted as due systematic variations in the formation time with system mass. This is predicted by the hierarchical clustering model of structure formation, with low mass systems collapsing first and then clustering together to form progressively larger structures. The exact process by which this affects the dark matter concentration is a matter of some uncertainty, but an interesting hypothesis is that halos all form with the same concentration, and subsequent accretion increases the virial radius and therefore the concentration (Salvador-Solé et al. 1998). Therefore halos that form at high redshift and therefore have had a long time to accrete material will have the highest concentrations. As the gravitational potential affects the temperature structure of the ICM, variation of the dark matter concentration would be expected to have an impact on the observed properties of the ICM.

We therefore have two processes which are observed to break self-similarity in galaxy clusters. Heating of the ICM which results in flattening of the gas density profile, and systematic variation in the dark matter halo concentration. In order to put constraints on these processes one useful approach is to construct models of galaxy clusters and then compare the predictions of the models with observed properties of clusters. A similar approach has previously been used by a number of authors (Cavaliere et al. 1997; Wu et al. 1998, 2000; Cavaliere et al. 1999; Balogh et al. 1999; Tozzi & Norman 2001; Bower et al. 2001) to explore the effects of energy injection. However these models have in general been driven from a theoretical perspective and have also tended to concentrate their efforts on predicting the evolution of cluster properties with redshift. Our aim is to construct a model of galaxy clusters, base on empirical results wherever possible, which will predict their properties at zero redshift. We will then use this model to investigate in detail how processes that break self-similarity, such as injection of energy into the ICM, affect the systems' observed properties. Comparison of the model predictions with observed data should then allow constraints to be placed on the various process involved in breaking self-similarity.

2 CLUSTER MODEL

In order to construct useful models of galaxy clusters to be compared with observations, a number of simplifying assumptions need to be made. In this case we will use only one-dimensional spherically symmetric models containing only dark matter and hot gas in hydrostatic equilibrium. While these approximations are crude, it should be noted that most analyses of observations of galaxy clusters involve the assumption of spherical symmetry. The approximation is fairly good for relaxed systems but it should be born in mind when considering the results that it is not a very good approximation to morphologically disturbed systems. In our model we make no distinction between dark matter and galaxies. However to a first approximation galaxies can be considered to be collisionless and while dynamical friction may be have some effect on the galaxies in low mass systems, it is unlikely that this will significantly affect the gravitational potential given that the galaxies make up only a small fraction of the mass of the system. Our approach is not to model the evolution of galaxy clusters to the state we observe them at present, but to model a variety of possible end points of cluster evolution and then compare them against observed clusters.

The primary parameter in the cluster model is the cluster mass. There are a number of possible ways the cluster mass could be defined. In general the mass within the region around the cluster centre that is in virial equilibrium is considered. The overdensity of this region with respect to the critical density, ρ_{crit}, can be calculated for the collapse of a spherical top-hat density perturbation (Peebles 1980). For a critical density universe this overdensity is $18\pi^2$ (~ 178) but can be significantly smaller for other cosmologies (Bryan & Norman 1998). An overdensity value of 200 is often used to define the outer boundary of a system as this is smaller than the virial radius for all reasonable cosmologies. In order for our results to be easily comparable with observations this overdensity was used. This results in a relationship between the mass, M_{200}, and the radius, R_{200}, of

$$R_{200} = \left(\frac{3M_{200}}{800\pi\rho_{crit}} \right)^{\frac{1}{3}}, \qquad (1)$$

which depends only on the critical density, ρ_{crit}. Throughout the modeling we adopt H_0=50 km s^{-1} Mpc^{-1} and $q_0 = 0.5$.

2.1 ICM density profile

In our model the gas density of the ICM is represented by the usual parameterization of the form,

$$\rho(r) = \rho(0) \left[1 + \left(\frac{r}{r_c} \right)^2 \right]^{-\frac{3}{2}\beta}, \qquad (2)$$

where r_c is the core radius, β is the density index and $\rho(0)$ is the density normalization. Jones & Forman (1984) have found this to be a good representation of the structure of the ICM in galaxy clusters. Vikhlinin et al. (1999), using the best quality ROSAT data, agree with this, although they find marginal evidence for steepening of the surface brightness profile at large radius. The fiducial value for the β parameter in our model was the canonical value of $\frac{2}{3}$. A fiducial gas fraction within R_{200} of 0.2, as observed by Lloyd-Davies & Ponman (2002), was used to set the normalisation of the gas density profile.

To define the core radius of the gas density profile, r_c, we use the value of 7 percent of R_{200} found by Lloyd-Davies & Ponman (2002) for systems where a cooling flow does not obscure the core. Selecting a value for the normalization of the gas density profile, $\rho(0)$, is not a trivial problem. One possibility is to normalize the profile such that the gas mass within R_{200} is $f_{gas}M_{200}$ where f_{gas} is the gas fraction of the cluster and a parameter to be specified. However this method may be unphysical as it does not allow gas to be pushed outside R_{200} when the specific energy of the gas is raised by a large amount. The normalisation of the gas density is therefore a boundary condition for the problem which needs careful consideration.

2.2 Dark matter density profile and gravitational potential

In our model no distinction between dark matter and stars is made, since both are expected to be collisionless on a large scale and neither will contribute significant emission to the X-ray data we will be comparing the model against. The dark matter distribution in our model is represented by a profile derived from numerical simulations (Navarro et al. 1995) of the form,

$$\rho_{DM}(r) = \bar{\rho}_{DM} \left[x(1+x)^2 \right]^{-1}, \qquad (3)$$

where $x = r/r_s$ and r_s is a scale radius. It should be noted that it has recently been suggested (Moore et al. 1999) that higher resolution simulations produce dark matter profiles with steeper central cusps, with asymptotic slopes of about 1.5 rather than 1 in the profile of (Navarro et al. 1995). However this effect is only important at small radii and as many observational results are derived using the profile of (Navarro et al. 1995) it is better suited to our purposes. It is possible to parameterize the concentration of the dark matter profile by the concentration parameter, $c = r_{200}/r_s$(Navarro et al. 1997). This concentration is seen to be anti-correlated with cluster mass in numerical simulations (Navarro et al. 1997) and this has been observed using X-ray data (Wu & Xue 2000; Sato et al. 2000; Lloyd-Davies & Ponman 2002). It is possible for us to take account of this variation in concentration using an analytical relation, for instance the relation

$$c = 34.9 \left(\frac{M_{200}}{10^{13} M_\odot} \right)^{-0.51}, \qquad (4)$$

is measured by Lloyd-Davies & Ponman (2002) for a sample of 20 galaxy clusters and groups. It should be noted that this relation is a rough characterization of a trend that shows considerable scatter. It is also possible to use the results of numerical simulations to define the dark halo concentration. The simulations of Navarro et al. (1997) predict concentration variation for a ΛCDM cosmology that can be approximated by the relation

$$c = 13.2 \left(\frac{M_{200}}{10^{13} M_\odot} \right)^{-0.20}, \qquad (5)$$

over the mass range of galaxy clusters and groups. This predicted relation is considerably flatter than the those observed by Wu & Xue (2000); Sato et al. (2000); Lloyd-Davies & Ponman (2002). We will investigate the use of both relations. The normalization of the of the dark matter profile, $\rho_{\bar{D}M}$, is set so that the cluster contains a dark matter mass of $(1 - f_{gas})M_{200}$ within R_{200}.

With the gas and dark matter profiles defined it is then possible to calculate the cluster temperature profile assuming hydrostatic equilibrium. For hydrostatic equilibrium and spherical symmetry, the equation

$$M(r) = -\frac{T(r)r}{G\mu} \left[\frac{dln\rho}{dlnr} + \frac{dlnT}{dlnr} \right] \qquad (6)$$

is satisfied (Fabricant et al. 1984). Given the mass and gas density profiles of a cluster the temperature profile is defined if the temperature is specified at some point. This introduces a second boundary condition to the problem that we must consider.

2.3 Effects of ICM cooling

In our model we do not explicitly model the effects of radiative cooling on the ICM for several good reasons. The amount of cooling in a particular cluster is will be related to the amount of time it has been left undisturbed by a major merger. This is an essentially random factor which varies from system to system, depending on its particular history. In contrast, our model aims to represent the present state of clusters, and to avoid the complications and uncertainties associated with cluster evolution. Secondly while the physics associated with radiative losses from ion-electron interactions in the ICM is well understood, the macroscopic effects of this cooling are not. The cooling is thought to result in a highly multiphase structure in the ICM but insufficient details are known to accurately model it. Thirdly, current observations do not provide enough information to provide a secure empirically based model of cooling in clusters.

We therefore adopt an approach of modeling the clusters without any cooling included and then attempting to take account of this when comparisons are made with ob-

servational data. In many cases it is possible to use observational data that has been corrected for the effects of cooling. This should allow a reasonably unbiased comparison of the model predictions with the data. In some cases, especially for galaxy groups cooling corrected data is not available. In these cases we will attempt to estimate what the likely effects of this are on our results using empirical data on correction for cooling. While this is far from ideal we believe that attempting to model the effects of cooling would introduce at least a similar amount of uncertainty into our results, given our present state of knowledge of cooling flows.

2.4 Boundary conditions

For a cluster that has the specific energy of the gas raised above the default value by flattening its gas density profile, there are two boundary conditions that must be specified. A normalisation for the gas density profile and a normalisation for the temperature profile. The default clusters need only one boundary condition, the temperature profile normalization, as the gas density normalisation is set by the gas fraction within the virial radius. However for the clusters with raise specific energy if the model is to allow the gas fraction to vary another constraint must be found. The most physically justifiable places to set the boundary conditions are at the centre of the systems and at the shock radius. However the position of the shock will be dependent on the amount of time that has elapsed since the system formed and also on the extent to which the infalling gas has been previously heated. Simulations suggest that the shock radius should occur at $1-1.5R_{vir}$ (Knight & Ponman 1997; Tozzi & Norman 2001) and that preheating can result in the shock propagating out as far as $2.5R_{vir}$ Tozzi & Norman (2001), since higher entropy gas has a higher sound speed.

One possible way of setting the density boundary condition is to use the observation of Lloyd-Davies & Ponman (2002) that the gas density extrapolated to R_{200} appears to converge to an approximately constant value for systems over a wide range of system masses. It should be noted that this result relies on extrapolating models fitted to data within a radius smaller than R_{200} and is not a direct measurement of the density at R_{200}. The gas density at R_{200} in the raised specific energy case can therefore be fixed at the density in the default case. This approach has the advantage of being extremely simple and has a least some justification from observations. We will therefore use this condition for the main results in the paper but will investigate the effect of alternative assumptions in Section 4.5.

A constraint is also needed to set the temperature boundary condition for the model. Unfortunately observed temperature profiles of galaxy clusters are much less well constrained than gas density profiles and therefore there are not much in the way of observational constraints on the temperature at R_{200}. Therefore it is necessary to resort to theoretical constraints on the temperature profile. A constraint Bower et al. (2001) derived from the numerical simulations of Eke et al. (1998) and Frenk et al. (2000) is that the temperature at the virial radius is approximately $0.5T_{vir}$, where T_{vir} is defined by the equation,

$$T_{vir} = \frac{\mu G M_{200}}{2kR_{200}}. \tag{7}$$

The mean mass per particle μ, is taken to be 0.6 amu.

2.5 Energy calculation

Observational studies (Lloyd-Davies & Ponman 2002; Helsdon & Ponman 2000b) suggest that the ICM density profiles in low mass systems are flattened compared to the profiles of high mass systems and this is observed as a reduction of the β parameter in low mass systems. There is little evidence to suggest, although it cannot be ruled out at present, that there is any significant effect on the core radius of the gas density profile. Figure 1 shows the core radii of the 8 galaxy cluster and groups in the sample of Lloyd-Davies & Ponman (2002) with reliable core radii measurements, plotted against emission weighted temperature. The solid line shows the relation predicted by the model between core radius and emission weighted temperature when the core radius is a constant fraction, 7 percent, of R_{200}. It can be seen that while the constraints of the data are not very strong, a model where the core radius is a fixed fraction of R_{200} appears to be quite consistent with the data. Since observational evidence strongly favours variation in the β parameter rather than the core radius (Lloyd-Davies & Ponman 2002; Helsdon & Ponman 2000b; Mohr et al. 1999), this approach therefore seems the most justifiable to take in modeling variation in the gas specific energy.

Our model therefore allows the gas specific energy to be raised by reducing the value of the β parameter and so flattening the gas density profile. In order to quantify difference in gas specific energy from the default case, a prescription is needed for how the total energy of the ICM is to be measured. It is possible given the temperature and density structure of the ICM, and also the dark matter distribution, to calculate the thermal and gravitational potential of the ICM. However it has been pointed out by Lloyd-Davies et al. (2000) that care must be taken in selecting the regions over which to calculate the energy as if the gas distribution is changed the energy of the ICM within a fixed radius will not be comparing the same mass of gas. To calculate the difference in the energy of the ICM between raised specific energy and default models we therefore calculate the difference between the total energy of the gas within R_{200} in the raised specific energy model and the total energy of the same mass of gas in the default model, which will be contained within a smaller radius. Altering the value of β affects both the gravitational potential and thermal energy of the gas, since the gas temperature profile will be modified if the gas density profile is changed. Using this prescription it is possible to alter the value of β until the desired difference in specific energy is achieved. This approach is similar to that used in the model of Bower et al. (2001).

3 OBSERVATIONAL DATA

In order to constrain the parameters of our cluster models observational data is needed to compare with the models predictions. There are a number of observed relations which the model should reproduce in order to be an accurate representation of galaxy clusters. For instance the mass temperature relation is a fundamental relation which the model will

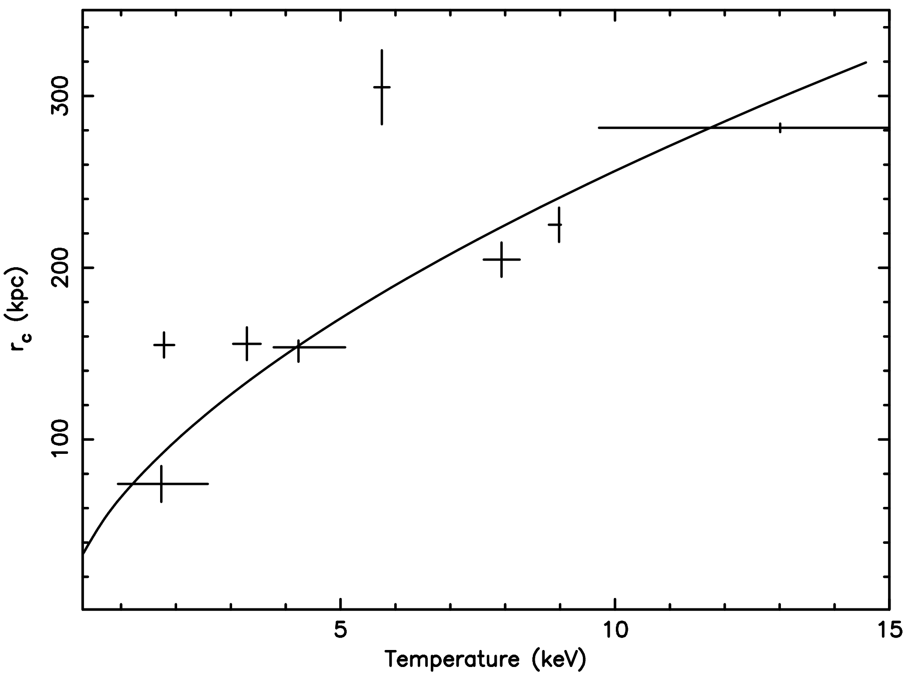

Figure 1. Core radius plotted against emission weighted temperature for the 8 galaxy clusters and groups in the sample of Lloyd-Davies & Ponman (2002) with reliable core radii measurements. The solid line shows the relation predicted by the model between core radius and emission weighted temperature when the core radius is a constant fraction, 7 percent, of R_{200}.

need to reproduce. The mass-temperature relation of Lloyd-Davies & Ponman (2002) for 20 systems over a large mass range was used to compare with the model predictions. The masses were derived from analytical models fitted to X-ray spectral images and the temperatures are cooling flow corrected. This is advantageous as our model does not model the effects of radiative cooling in cluster cores.

An observed β-temperature relation to compare against our model predictions were obtained from Lloyd-Davies & Ponman (2002) (20 systems) and Helsdon & Ponman (2000b) (6 systems). Lloyd-Davies & Ponman (2002) fitted spherically symmetric gas density profiles to X-ray spectral images of galaxy clusters and groups while parameterizing either the gas temperature profile or dark matter distribution by some functional form. Helsdon & Ponman (2000b) fitted surface brightness profiles to X-ray images of galaxy groups while allowing ellipticity. Their β values are therefore only strictly comparable to the gas density profiles of Lloyd-Davies & Ponman (2002) on the assumption of isothermality. Also the model fitting of Lloyd-Davies & Ponman (2002) assumes spherical symmetry whereas Helsdon & Ponman (2000b) do not, and the temperatures of Lloyd-Davies & Ponman (2002) are cooling flow corrected whereas those of Helsdon & Ponman (2000b) are not. Despite these differences the results obtained for these to studies are remarkably similar.

The luminosity-temperature relation data we use to compare with our model predictions is taken from Markevitch (1998) and Arnaud & Evrard (1999) for galaxy clusters and Helsdon & Ponman (2000a) for galaxy groups. The data of Markevitch (1998) have been corrected for the effects of cooling and the sample of Arnaud & Evrard (1999) was chosen not to contain any systems significantly affected by cooling. However the data of Helsdon & Ponman (2000a) has not been corrected for the effects of cooling and there is not much other data available for these low temperature systems. Therefore when comparing them with the model predictions we will attempt to quantify the likely effects on our results of the galaxy groups being uncorrected. It should also be noted that the radii from within which luminosity data are extracted are not consistent. This is more of a problem for the low mass systems which have flatter surface brightness profiles. In these systems the luminosity has generally been derived within approximately 0.3 R_{200} whereas for more massive system the luminosity derived within some larger radius.

4 RESULTS

A fundamental parameter in all the relations we are interested in is the gas temperature. We therefore consider how best to extract a gas temperature from our model and then go on to compare the predictions of the model with the relations listed in Section 3. Finally we will investigate how the boundary conditions affect the results in Section 4.5.

4.1 Temperature derivation

The effects of raising the specific energy of the gas or increasing the dark matter concentration on the gas temperature are quite similar. In both cases the central temperature and temperature gradient are increased. The effects of increasing the dark matter concentration are more centrally concentrated however, with most of the temperature increase occurring near the core of the system. In contrast raising

the specific energy of the gas raises the gas temperature out to quite large radii. In terms of the effect on the emission-weighted temperature increasing the dark matter concentration is fairly straight forward, since it has no effect on the gas density profile and hence the emission-weighting is not significantly altered. Increasing the dark matter concentration therefore results in an increase in the emission-weighted temperature. In the case of raising the specific energy of the gas the flatter gas density profiles result in the emission being weighted more towards larger radii where the gas temperature is lower. The effect on the emission-weighted temperature is therefore not simple and in general depends on the radius within which the emission-weighted temperature is calculated. For temperatures calculated within R_{200} raising the specific energy of the gas has little or no effect whereas temperatures calculated within 0.3 R_{200} rise as the specific energy of the gas is raised.

Perhaps the most natural way to derive gas temperatures from our model is to calculate emission-weighted temperatures within R_{200} since the the observed temperatures that we will be using will be weighted in this way. However it should be noted that most actual observations of galaxy clusters do not extend to this radius. In the case of the low mass systems that we are particularly interested in, 0.3 R_{200} is more representative of the radii within which emission-weighted temperatures are derived. Even in high mass systems the X-ray data rarely extend out to anywhere near R_{200}. We therefore use 0.3 R_{200} as the radius within which we extract emission-weighted temperatures. It should be noted that in general the radius to which X-ray data are available increases with increasing system temperature and this may lead to some bias in the observed data. As previously noted the model does not contain any cooling whereas in many observed systems cooling has some effect on their emission-weighted temperatures. In some cases it is possible to obtain data corrected for the effects of cooling and where possible we make use of it.

4.2 Mass-temperature relation

An important relation which will constrain our models is the mass temperature relation. Raising the specific energy of the ICM can affect its temperature, and as the flattening of the density profile can push gas outside the virial radius, the mass within R_{200} can also be affected to a certain extent. Self-similar theory predicts that the mass should be proportional to $T^{\frac{3}{2}}$ and this is generally seen in numerical simulations (Navarro et al. 1995). Figure 2 shows the total mass within R_{200} plotted against emission weighted temperature within 0.3 R_{200} for a range of cluster masses. The systems all have a constant concentration parameter of 10. To compare our simulated mass-temperature relations to observations, we use data from Lloyd-Davies & Ponman (2002) for their sample of 20 galaxy clusters and groups. The solid line shows the mass-temperature relation for the default model. As expected from self-similar scaling the relation is a powerlaw with an index of 1.5. The other lines show the relations for systems where the specific energy has been raised by 0.25 keV per particle (dot-dashed), 0.5 keV per particle (dashed) and 0.75 keV per particle (dotted). It can be seen that the amount of deviation from the self-similar mass-temperature relation is not large, even when the specific energy of the gas is raised by a large amount. It should be noted that in the 0.75 keV per particle (dotted) case line terminates before the lowest mass is reached as beta is flattened to 0 at this point. It therefore seems that large deviations from the mass-temperature relation due to heating are not possible, at least with our model.

The other process that might affect the mass temperature relation is the variation in dark matter concentration. The concentration will not affect the mass within R_{200} but it will have some effect on the temperature. Figure 3 shows the total mass within R_{200} plotted against emission weighted temperature within 0.3 R_{200} for several values of the concentration parameter. It can be seen that as the concentration increases from c=2 (solid line) to c=20 (dotted line) the normalisation, defined as the mass at 1 keV, decreases from $1.5 \times 10^{14} M_{\odot}$ to $3 \times 10^{13} M_{\odot}$. This can be compared with other relations from observations and theory. The NFW relation obtained from numerical simulations has a normalisation of $8.7 \times 10^{13} M_{\odot}$ (Navarro et al. 1995) which is in between our two extreme relations. Lloyd-Davies & Ponman (2002) have fitted a $T^{\frac{3}{2}}$ powerlaw to this sample of 20 galaxy clusters and groups and obtained a normalisation of $9.7 \pm 1.4 \times 10^{13} M_{\odot}$ which is also bracketed by our simulated relations. However this observed normalisation is for all galaxy systems. Lloyd-Davies & Ponman (2002) found that for systems with temperatures below 4 keV the normalisation was $3.1 \pm 0.4 \times 10^{13} M_{\odot}$ which is comparable with our high concentration model relations. This can be understood in terms of the variation in the concentration parameter with system mass. We have seen in Figure 3 that the dark matter concentration has a direct effect on the normalisation of the mass-temperature relation. Since lower mass systems are more concentrated, on average, their mass-temperature relation would be expected to have a lower normalisation. Whereas rich clusters of galaxies with concentration parameters of around 5 would be expected to have a much higher normalisation to their mass-temperature relation, which is what Lloyd-Davies & Ponman (2002) observe. The actual mean mass-temperature relation should therefore be steeper than the simple $T^{\frac{3}{2}}$ relation expected from self-similar scaling, due to the increase in concentration in lower mass systems. This effect is indeed observed by Lloyd-Davies & Ponman (2002) who observe the relation to have a logarithmic slope of 1.96 ± 0.21.

Figure 4 shows the data with overlayed simulated mass temperature relations for different dark matter concentration variation prescriptions. The solid line shows the simulated relation with the concentration parameter varying with system mass as specified by Equation 4. It can be seen that this relation is much steeper than the self-similar $T^{\frac{3}{2}}$ relation (dotted line), with a logarithmic slope of ~ 2. The relation appears to be a reasonable fit to the observed data, although if anything it is slightly steeper. The dashed line shows the simulated relation with the concentration parameter varying with system mass as predicted by Navarro et al. (1997) for a ΛCDM cosmology (see Equation 5). This relation is flatter than that predicted by Equation 4, with a logarithmic slope of ~ 1.75, somewhat flatter than is generally observed (Sato et al. 2000; Nevalainen et al. 2000; Lloyd-Davies & Ponman 2002). It therefore seems that observations generally favour

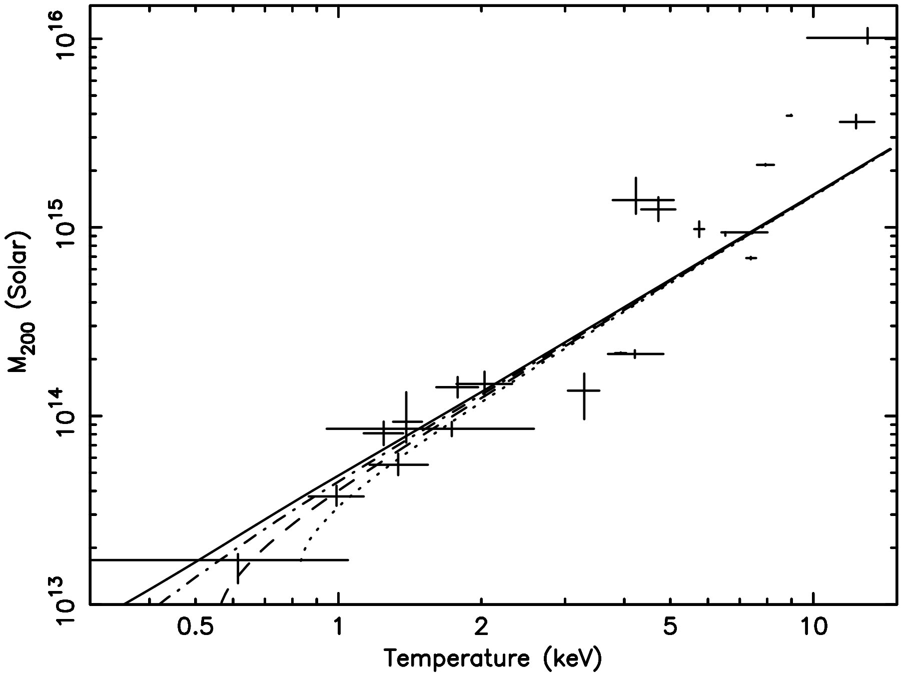

Figure 2. Total mass within R_{200} against emission weighted temperature within 0.3 R_{200} for simulated relations with the specific gas energy raised various amounts; 0.0 keV per particle (solid); 0.25 keV per particle (dot-dashed); 0.5 keV per particle (dashed); 0.75 keV per particle (dotted). A constant concentration parameter of 10 is used for all the relations. Observed data (crosses) are taken from the sample of Lloyd-Davies & Ponman (2002).

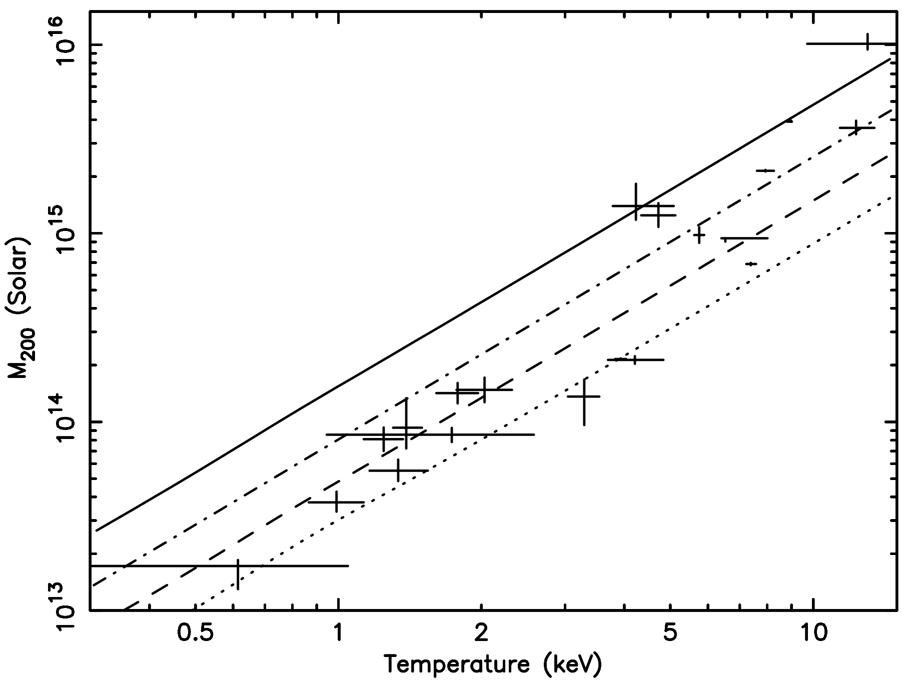

Figure 3. Total mass within R_{200} against emission weighted temperature within 0.3 R_{200} for simulated relations with various dark matter concentrations; c=2 (solid); c=5 (dot-dashed); c=10 (dashed); c=20 (dotted). The excess energy for all the relations is zero. Observed data (crosses) are taken from the sample ofLloyd-Davies & Ponman (2002).

concentration variation that is steeper than the ΛCDM prediction. For this reason the observed relation Equation 4 will used to derive the following the results but comparisons with the ΛCDM prediction will be included where necessary. The mass-temperature relation appears to be significantly affected by the variation in concentration of galaxy clusters and these effects should propagate into other relations involving galaxy cluster temperatures.

4.3 Beta-temperature relation

The relationship between the asymptotic slope of gas density profile and the temperature of galaxy clusters is important since it appears that the main result of raising the specific energy of the ICM is to flatten the gas density profile (Ponman et al. 1999; Lloyd-Davies & Ponman 2002). Since we have previously seen that variation in the dark matter concentration of galaxy clusters can affect their emission-weighted temperatures, some effect on the beta-temperature relation is also to be expected. Figure 5 shows β plotted against emission weighted temperature within 0.3 R_{200} for various dark matter concentration parameters. The model predictions are shown for concentration parameters of c=10 (solid), c=15 (dot-dashed), c=20 (dashed) and c=25 (dotted). In each case the specific energy of the ICM has been raised by 0.75 keV per particle. The observed data are taken from the samples of Helsdon & Ponman (2000b) (circles) and Lloyd-Davies & Ponman (2002) (crosses). It can be seen that as the concentration is increased, the relation is pushed to higher temperatures. However the change in the relation is not particularly large, and cannot entirely explain the scatter in $\beta - T$ observed.

The other parameter that can be varied is the amount by which the specific energy of the ICM is raised. Figure 6 shows β plotted against emission weighted temperature within 0.3 R_{200} with the dark matter concentration varying according to Equation 4. The model predictions for the specific gas energy raised by 0.25 keV per particle (solid), 0.5 keV per particle (dot-dashed), 0.75 keV per particle (dashed) and 1.0 keV per particle (dotted) are shown. It can be seen that there appears to be a considerable amount of scatter with none of the energy injection values chosen providing a good fit to all the data. In fact the statistical errors on the points accounting for only 5 percent of the scatter about a mean fitted relation. This result is not significantly changed by removing the Helsdon & Ponman (2000b) points. If this scatter is interpreted as due to a scatter in the amount of energy injected into each system then energy injection covering at the range 0.25 and 1.0 keV per particle are required to match the data. Only a small component of the scatter can be attributed to scatter in the concentration parameters of the systems.

It is possible to use the predictions of the model to derive excess energies for individual low mass systems. Since the dark halo concentration parameters of the systems affect the model predictions to some extent, these measurements are best done for systems with known dark matter concentrations. The sample of Lloyd-Davies & Ponman (2002) is the only data available where concentration parameters have been measured for low mass systems and also has the advantage of the various observational parameters being derived within a consistent set of radii. Excess energies were therefore derived from the β parameters and temperatures of the eight systems in the sample of Lloyd-Davies & Ponman (2002) with temperatures below 3 keV and are shown in Figure 7 plotted against the systems emission weighted temperatures within 0.3 R_{200}. It can be seen that apart from the three lowest temperature systems the there is little trend in excess energy with temperature although there is considerable scatter. The mean excess energy is ∼1 keV per particle excluding the three lowest temperature points. It is clear that the three lowest temperature systems fall considerably below this value and show a strong trend of decreasing excess energy with temperature. This trend is also noticeable in Figure 6. While it is possible that this is a real effect, the role of selection effect in this trend should not be discounted.

In particular the luminosity of a system decreases as excess energy increases and this effect is much more pronounced in low temperature systems. For this reason if a sample is flux limited and there is some distribution in excess energies, then there will be a tendency for systems with low excess energies to be preferentially selected. The magnitude of this effect will increase with decreasing system temperature since the range of luminosities for a given range of excess energies is larger for lower mass systems. The three lowest temperature systems may therefore represent only the extreme low end of the distribution in excess energies since low temperature systems with higher excess energies would not have high enough luminosities to be part of the sample of Lloyd-Davies & Ponman (2002). It should be noted that one of the five higher temperature systems has an excess energy higher than the mean by an amount similar to the amount that the three low temperature systems are lower than the mean. This suggests that there may be enough intrinsic scatter in the excess energies of the systems to account for this effect.

We can approximate where this flux limit will lie in Figure 7 assuming that all these low mass systems are at approximately the same redshift and there is some limiting luminosity below which they cannot be observed. Helsdon & Ponman (2000b) provide an extensive sample of X-ray bright galaxy groups all of which are brighter than 2×10^{41} erg s^{-1}. Using out model a line of constant luminosity in the excess energy-temperature plane can be calculated by adjusting the excess energy until that luminosity is achieved for a range of system masses. This luminosity limit of 2×10^{41} erg s^{-1} is shown in Figure 7 as the dashed line. It can be seen that in general in the sample fall below the line where there luminosities should be larger than 2×10^{41} erg s^{-1}. The drop in excess energy for the lowest mass systems also appears to shadow the luminosity limit in the expected way. It therefore seems quite likely that this is a selection effect rather than a intrinsic property of low mass galaxy groups.

4.4 Luminosity-temperature relation

A well studied relation between galaxy cluster properties is the luminosity-temperature relation. This is primarily because the luminosity and temperature are the two most easily measured properties of galaxy clusters. For the highest redshift clusters, which are extremely important from a cosmological perspective, these are the only X-ray properties that can be measured. It has long been know that the luminosity-temperature relation does not follow the predic-

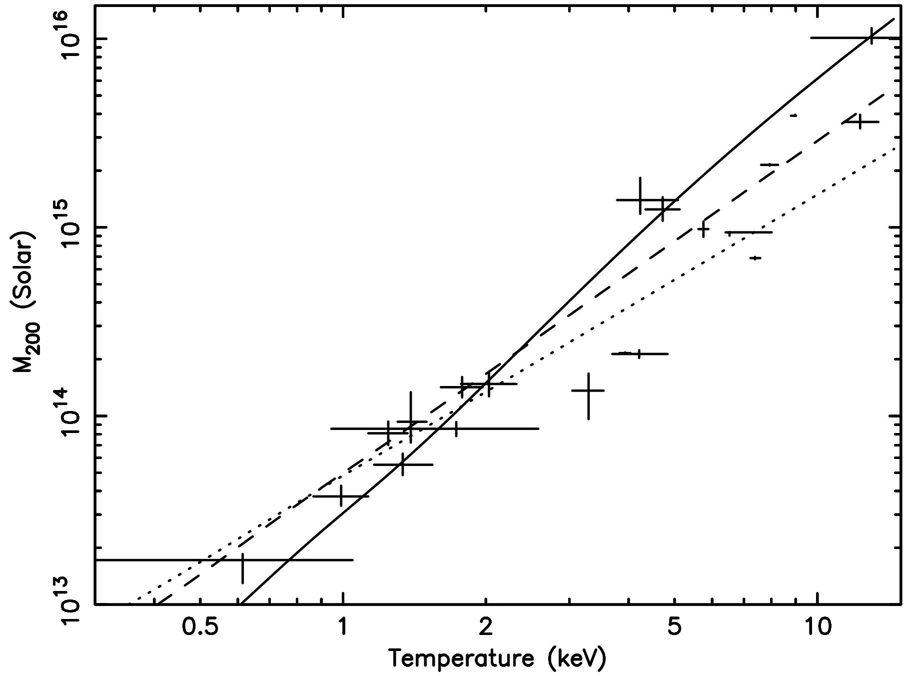

Figure 4. Total mass within R_{200} against emission weighted temperature within 0.3 R_{200} for simulated relations for dark matter concentration varying according to Equation 4 (solid), Equation 5 (dashed) and a constant c=10 (dotted). The excess energy for all the relations is zero. Observed data (crosses) are taken from the sample of Lloyd-Davies & Ponman (2002).

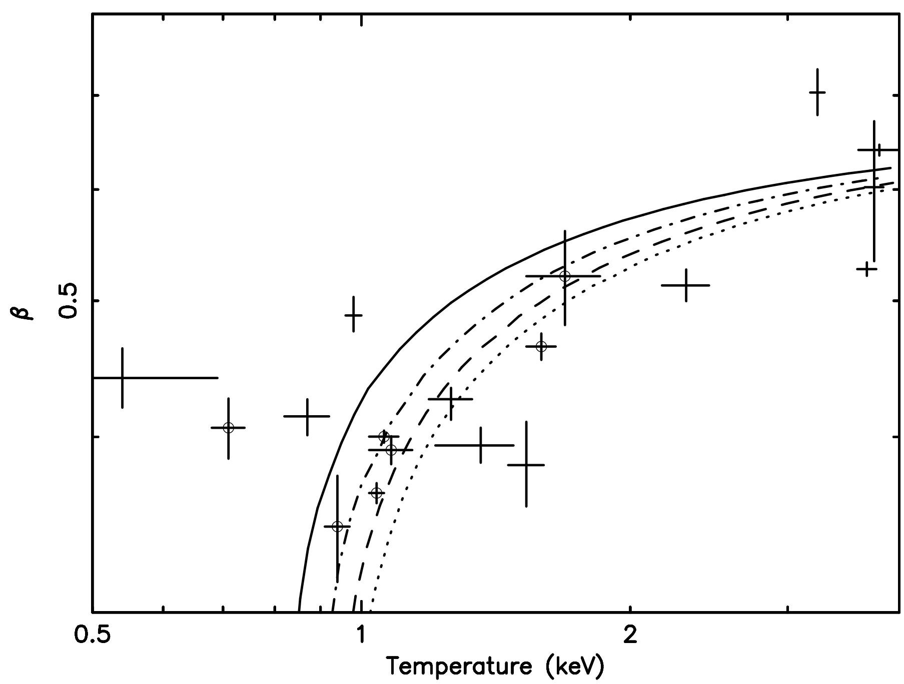

Figure 5. Asymptotic slope of the gas density profile, β, plotted against emission weighted temperature within 0.3 R_{200} for simulated relations with various dark matter concentrations; c=10 (solid), c=15 (dot-dashed), c=20 (dashed) and c=25 (dotted). Observed data are taken from the samples of Helsdon & Ponman (2000b) (circles) and Lloyd-Davies & Ponman (2002) (crosses).

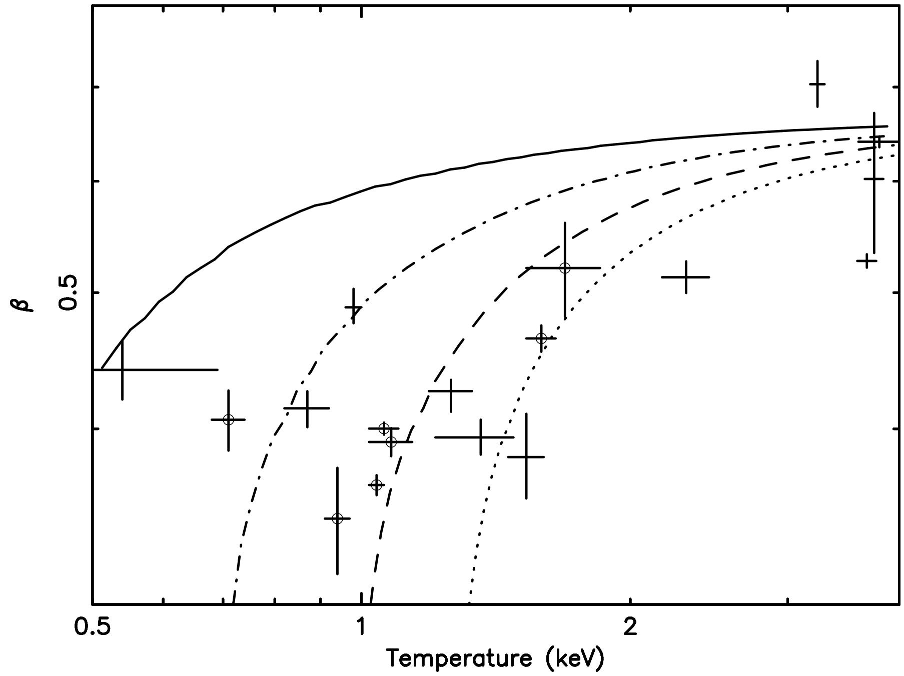

Figure 6. Asymptotic slope of the gas density profile, β, plotted against emission weighted temperature within 0.3 R_{200} for simulated relations with specific energies raised to 0.25 keV per particle (solid), 0.5 keV per particle (dot-dashed), 0.75 keV per particle (dashed) and 1.0 keV per particle (dotted). In all cases the dark matter concentration varies with mass according to the relation given in Equation 4. Observed data are taken from the samples of Helsdon & Ponman (2000b) (circles) and Lloyd-Davies & Ponman (2002) (crosses).

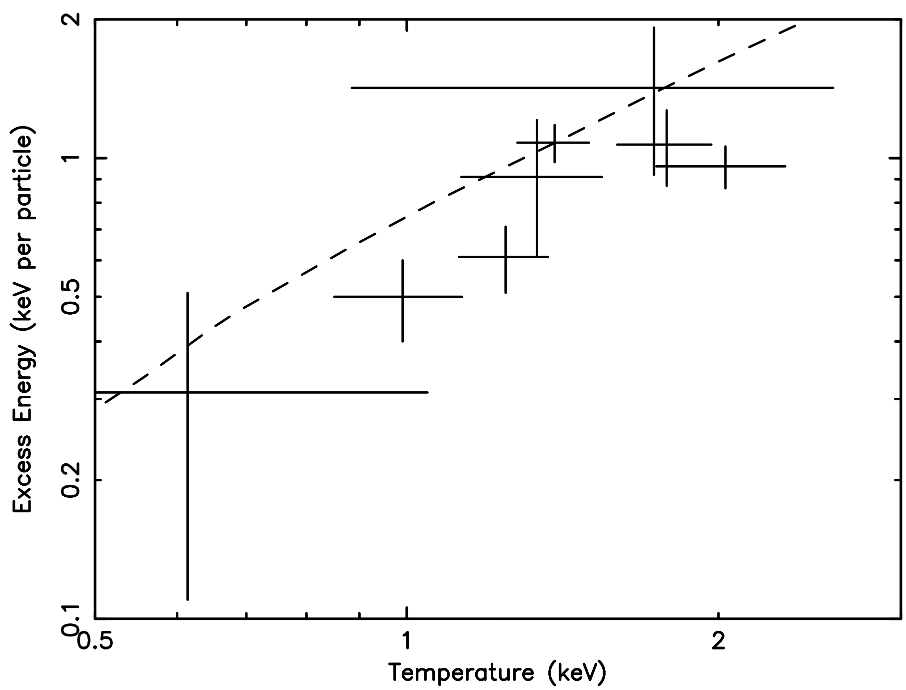

Figure 7. Excess gas energy within R_{200} plotted against emission weighted temperature within 0.3 R_{200} for the eight systems in the sample of Lloyd-Davies & Ponman (2002) with temperatures below 3 keV. The dashed line is the model prediction for a constant luminosity of 2×10^{41} erg s^{-1} (see text).

tions of self-similar theory. For self-similar clusters, luminosity would be expected to be proportional to temperature squared (Kaiser 1986). However observational studies of the luminosity-temperature relation tend to find steeper slopes, with luminosity typically proportional to temperature cubed. For instance Edge & Stewart (1991) have measure the luminosity-temperature relation for a sample of galaxy clusters to be $L \propto T^{2.62\pm0.10}$; David et al. (1993) found $T \propto L^{0.297\pm0.004}$ and White et al. (1997) found $T \propto L^{0.30\pm0.05}$. These results are all in the region of $L \propto T^3$. It has been suggested that this steepening from the predicted relation might be caused by cooling flows (Allen & Fabian 1998). However Markevitch (1998) has derived a luminosity-temperature relation corrected for the effects of cooling flows and measured a slope of 2.63±0.27. Arnaud & Evrard (1999) measured the slope of the luminosity-temperature relation for a sample galaxy clusters without strong cooling flows to be 2.88 ± 0.15 and Ettori et al. (2001) obtain a slope of 2.7 in the luminosity-temperature relation for a corrected sample of galaxy clusters. It therefore seems clear that cooling flows cannot provide an explanation for the steepening of the luminosity-temperature relation in galaxy clusters.

Various authors have extended this work down to lower mass systems, galaxy groups, and found that the luminosity-temperature relation steepens in these systems. Ponman et al. (1996) measured the slope of the luminosity-temperature relation for Hickson compact groups to be 8.2 ± 2.7 while Helsdon & Ponman (2000b) found the slope for a sample of loose groups to be 4.9 ± 0.8. Helsdon & Ponman (2000a) have combined a sample of loose and compact groups to derive a slope of 4.3 ± 0.5. This is significantly steeper than the luminosity-temperature relation for rich galaxy clusters. This steepening of the luminosity-temperature relation in galaxy groups has generally been attributed to the effect of heating by galaxy winds (Cavaliere et al. 1997; Helsdon & Ponman 2000b; Bower et al. 2001). There appear to be two separate effects on the luminosity-temperature relation which may or may not be related. The steeper than predicted slope for high mass systems and the steepening of the relation in low mass systems. It is therefore extremely interesting to ask how the luminosity-temperature relation predicted by our model compares with observations.

However in order to calculate the luminosity, some prescription is needed for the emissivity of a plasma of a given temperature, density and metallicity. We use the method of Knight & Ponman (1997), which uses bilinear interpolation over the tabulated cooling function of Raymond et al. (1976), to calculate the cluster luminosities and to derive the emission weighting for the emission weighted temperatures.

Figure 8 shows the bolometric luminosity within 0.3 R_{200} against emission weighted temperature within 0.3 R_{200}. The dotted line shows the model prediction for systems with a fixed dark matter concentration of c=10 and no heating of the ICM. This is compared with observed data for galaxy clusters from Markevitch (1998) (triangles) and Arnaud & Evrard (1999) (squares). The data of Markevitch (1998) is corrected for cooling and that of Arnaud & Evrard (1999) is selected to contain no systems significantly affected by cooling. Observed data for galaxy groups is taken from the sample of Helsdon & Ponman (2000b) (crosses) and is not corrected for cooling. It can be seen that the model relation is considerably flatter than the observed data. To investigate the effect of varying the concentration parameter on the luminosity-temperature relation, the relations for dark matter concentration varying according to Equations 5 and 4 are shown as the dashed and solid lines respectively. The effect of varying the concentration is to significantly steepen the luminosity-temperature relation, bringing it into much better agreement with the observed data. The relation with concentration varying according to Equation 5 has a logarithmic slope of ~ 2.2 above 2 keV, the one varying according to Equation 4 has a logarithmic slope of ~ 2.7 above 2 keV. It can be seen that there is some variation in the slope with system temperature. The relation with a constant concentration parameter of c=10 (dashed line) has a logarithmic slope of ~ 1.97 although again there is some variation in the slope with system temperature. It should be noted that strictly the expectation that the luminosity-temperature relation will have a logarithmic slope of 2 is only true if the emission process is pure thermal bremsstrahlung. The contribution of emission lines at low temperatures will tend to flatten the relation so that the relation will be expected only to asymptote to a logarithmic slope of 2 at high temperatures where the effect of emission lines is less important.

The normalisation of the luminosity-temperature relation for varying dark matter concentration in Figure 8 appears somewhat higher than the mean trend of the observed data. One possible reason for this is that the relation we use for the core radius in the model is taken from Lloyd-Davies & Ponman (2002), whose sample was picked to be as relaxed as possible. It is possible that the value of 7 percent of R_{200} which we use is not representative of galaxy cluster population as a whole and that less relaxed clusters have significantly larger core radii. Previous studies that did not limit themselves to relaxed systems, have measured core radii that are considerably larger (Mohr et al. 1999). Increasing the core radius for a fixed gas fraction will reduce the central gas density and therefore the luminosity. Another related point is that the canonical value of $\beta = \frac{2}{3}$ that we use as the default in the model when no heating has occurred does not have a great deal of theoretical support. If in fact the mean value of β in real high mass clusters never quite reaches the canonical value and this would also reduce the luminosity for a given gas mass. It should be noted that the mean β for the systems above 4 keV in the sample of Lloyd-Davies & Ponman (2002) is 0.61. It is also possible that the gas fraction of 0.2 that we are using is somewhat higher than the mean value in the systems to which we are comparing the model.

It can be seen in Figure 8 that while the variation of the concentration parameter results in the high temperature part of the relation being a reasonable fit to the data the low temperature part of the relation still over-predicts the luminosities of the low temperature systems. To investigate whether ICM heating can improve the fit for the low temperature systems relations were plotted with varying amounts of energy injection into the ICM. Figure 9 shows the luminosity-temperature relations for energy injection of 0.0 keV per particle (solid); 0.25 keV per particle (dot-dashed) 0.5 keV per particle (dashed) and 0.75 keV per particle (dotted). It can be seen that the heating the ICM by up to 0.75 keV per particle has little effect on the luminosity-temperature relation for the galaxy clusters. However the heating does have a significant effect on the

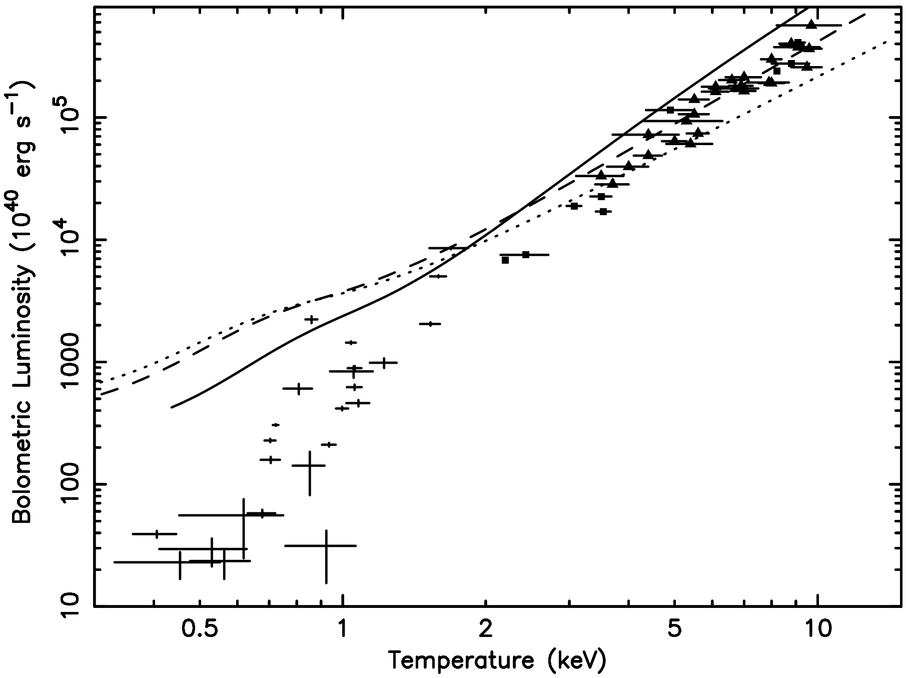

Figure 8. Bolometric luminosity within 0.3 R_{200} against emission weighted temperature within 0.3 R_{200} for simulated relations for dark matter concentration varying according to Equation 4 (solid), Equation 5 (dashed) and a constant c=10 (dotted). Observed cooling corrected data are shown for the sample of Markevitch (1998) (triangles). The sample of Arnaud & Evrard (1999) (squares) is uncorrected but was chosen not to contain strong cooling flows. The data shown as crosses is for the sample of Helsdon & Ponman (2000b) and is not cooling corrected.

luminosity-temperature relation for the galaxy groups. As is to be expected the flattening of the gas density profile reduces the luminosity of the low mass systems towards the observed relation. It appears that energy injection of between 0.25 and 0.5 keV per particle best reproduces the observed relation. It should be noted that there is considerable scatter in the observed luminosity-temperature relation in galaxy groups, but given the possibility of scatter in both the injected energy and concentration parameter this is not surprising.

One discrepancy in this analysis is that the galaxy group data is affected to some extent by cooling, while the model does not take cooling into account. The luminosity-temperature relation is particularly susceptible to the effects of cooling since both parameters will be affected by it. We have attempted to to quantify the effects of this on our results using the observed properties of cooling flows in groups. Helsdon & Ponman (2002) find that these contribute typically 25 percent of the group luminosity, whilst Lloyd-Davies & Ponman (2002) find that removing the effects of cooling from spatial-spectral models fitting group X-ray data results in an increase in mean temperature of about 20 percent. The estimated total average effect of correcting the galaxy group data for cooling is shown as an arrow in Figure 9. It can be seen that with this correction, most of the data would be consistent with a mean injection energy of between 0.5 and 0.75 keV per particle.

4.5 Effects of boundary conditions

The results presented so far are for a specific set of plausible boundary conditions, given our present knowledge. The temperature at R_{200} is fixed at $0.5T_{vir}$ where T_{vir} is defined by Equation 7 and the density at R_{200} is constant. However it is possible that these boundary conditions are over simplified. To test the effect of these assumptions the boundary conditions must be varied in order to see how much difference they make to the results. The temperature boundary condition of $0.5T_{vir}$ in particular is based only on numerical simulations rather than an empirical evidence. Not only is it possible that 0.5 is the wrong normalization but more importantly it is possible that it may not scale with T_{vir}. For example, if the intergalactic medium (IGM) has been heated before the cluster forms, in the absence of significant cooling the minimum temperature at R_{200} will be the temperature to which the IGM was heated. In reality this represents an upper limit to the temperature constraint, since if the IGM was heated a long time before the system collapsed, the Hubble expansion would subsequently lower the gas temperature. In low mass systems this constraint would result in the temperature at R_{200} having a lower limit of $\frac{2}{3}\Delta E$ where ΔE in the energy injection in keV per particle. To assess how altering to the temperature boundary condition might affect the results, β-temperature relations with and without a minimum temperature were simulated for an energy injection of 0.75 keV per particle and dark matter concentration varying according to Equation 4. This is shown in Figure 10. The solid line shows the relation with no minimum temperature and the dashed line shows the effect of imposing a minimum.

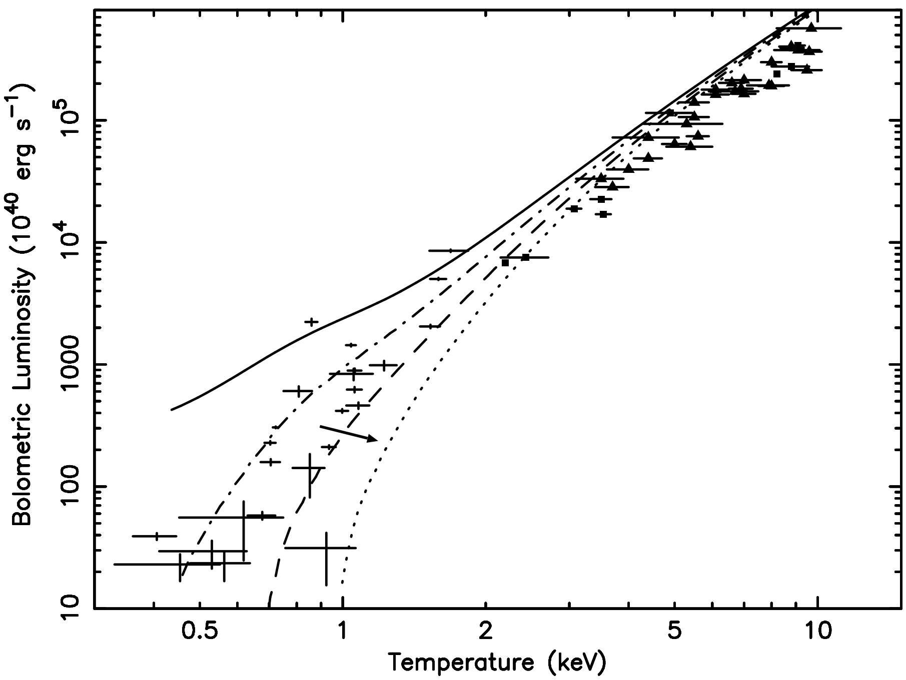

Figure 9. Bolometric luminosity within 0.3 R_{200} against emission weighted temperature within 0.3 R_{200} for simulated relations with excess gas energy of 0.0 keV per particle (solid); 0.25 keV per particle (dot-dashed) 0.5 keV per particle (dashed) and 0.75 keV per particle (dotted). In all cases the dark matter concentration varies with mass according to the relation given in Equation 4. Observed cooling corrected data are shown for the sample of Markevitch (1998) (triangles). The sample of Arnaud & Evrard (1999) (squares) is uncorrected but was chosen not to contain strong cooling flows. The data shown as crosses is for the sample of Helsdon & Ponman (2000b) and is not cooling corrected. The arrow shows is an estimate of the average effect on the Helsdon & Ponman (2000b) data if it was corrected for cooling.

It can be seen that the result of introducing a minimum temperature to the temperature boundary condition is to reduce the change in β that is needed, to get a specified excess energy, for systems below a certain emission-weighted temperature. In this case at $\sim$ 2 keV the temperature at R_{200} hits the limit of $\frac{2}{3}\Delta E$ and systems below this emission-weighted temperature have higher gas temperatures at R_{200} in the raised specific energy case than the default case. This causes more energy to go into heating the ICM and less into flattening the gas density profile resulting in higher values of β. Increasing the minimum temperature at the outer boundary of the model will increase the emission-weighted temperature below which the β parameter deviates from the predictions of our standard model. The mechanism by which the effect of energy injection on the boundary condition has been implemented is very crude, but it does give good indication of the general effect that this sort of process would have. The effect is to cause the decrease in β with decreasing temperature to become less drastic below a certain temperature. While the number of very low temperature systems in our sample is not that large, the data do suggest a steep decrease in β which flattens off below $\sim$ 1 keV. However as noted previously there are likely to be strong selection effects at work in the lowest mass system which might be expected to cause only the low mass systems with the highest β parameters to be observable.

5 DISCUSSION

We have modeled two major processes that result in the breaking of the expected self-similarity of galaxy clusters; energy injection and variation in the dark matter halo concentration. Both these processes are shown to have significant effects on the properties of galaxy clusters. The mass temperature relation is not significantly affected by heating of the ICM except for the lowest mass systems whose temperatures are increased. However variation of the dark matter concentration significantly alters the normalisation of the mass-temperature relation. The observed trend in concentration parameter with mass (Lloyd-Davies & Ponman 2002), results in a steepening of the mass-temperature relation from its expected logarithmic slope of 1.5 to $\sim$ 2. This is a good match the observed data, where slopes of this order are observed: 1.96 ± 0.21 (Lloyd-Davies & Ponman 2002), 1.79 ± 0.14 (Nevalainen et al. 2000) and 2.04 ± 0.04 (Sato et al. 2000). The observed mass-temperature relation therefore appears to be well matched by the model predictions when systematic variation in dark matter concentration with system mass is included.

The relation between the asymptotic slope of the gas density profile and the emission weighted temperature has generally been interpreted as due to energy injection into the ICM. The model produces relations that roughly follow the trend in the data for energy injection in the range 0.5 to 1.0 keV per particle. However the relation has a considerable amount of scatter and no one model appears to be consistent

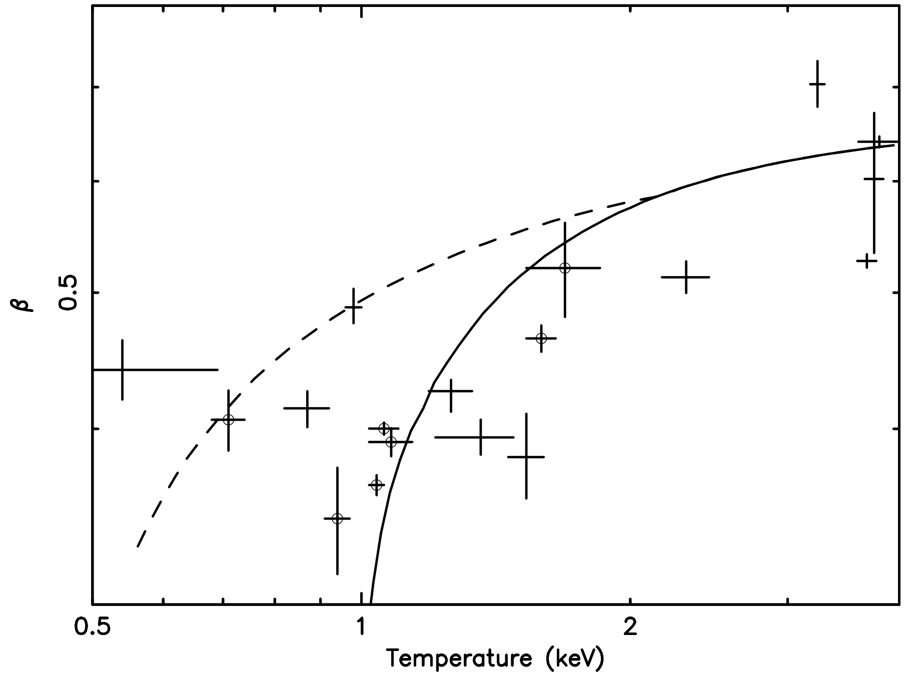

Figure 10. Asymptotic slope of the gas density profile, β, plotted against emission weighted temperature within 0.3 R_{200} for dark matter concentration varying according to the relation given in Equation 4 and raised specific energy of 0.75 keV per particle. The solid line is the relation for the temperature boundary condition of $0.5T_{vir}$. The dashed line is the relation when a minimum temperature of $\frac{2}{3}\Delta E$ is imposed on the boundary condition. Observed data are taken from the samples of Helsdon & Ponman (2000b) (circles) and Lloyd-Davies & Ponman (2002) (crosses).

with all the data. This may be due to real scatter in the amount of excess energy injected into the systems, although it is possible that the errors on the observational data have been underestimated. The effect of varying the dark matter concentration is to push the relation to higher temperature. The inclusion of systematic variation of the concentration parameter with system mass as observed by (Lloyd-Davies & Ponman 2002) has the effect of slightly lowering the amount of energy that in needed to get a given β value. Scatter in the concentration of dark matter halos may make some contribution to the scatter in the beta-temperature relation. However it does not appear to be a large enough effect to be the dominant cause.

The luminosity-temperature relation of the model has the expected logarithmic slope of ~ 2 when the model is self-similar. However introducing the systematic variation in dark matter concentration observed by Lloyd-Davies & Ponman (2002) steepens the slope to 2.7 which is comparable with the slopes of 2.6 to 2.9 observed by recent authors (Markevitch 1998; Arnaud & Evrard 1999; Ettori et al. 2001). This is to be expected as increasing the concentration of the low mass systems will increase their emission weighted temperatures and so steepen the relation. It therefore seems clear that the steepening of the slope of the luminosity-temperature relation in the cluster regime is largely, if not entirely, explained by the systematic variation of dark matter halo concentration with system mass. It should also be noted that the systematic trend seen by Lloyd-Davies & Ponman (2002) has considerable scatter which is expected theoretically as low mass systems are expected to form a higher redshift only on average. Scatter in the concentration of dark matter halos could contribute a considerable amount to the scatter in the luminosity-temperature relation. However cooling flows are also likely to contribute a considerably component to the scatter.

Models including systematic variation of the dark matter concentration with system mass reproduces the slope of the luminosity-temperature relation for galaxy clusters but does not predict the steepening of the relation seen in galaxy groups. However the addition of heating of the ICM steepens the relation in galaxy groups as flattening the gas density profile reduces the luminosity and increases the emission weighted temperature. There is considerable scatter in the observed relation but most systems appear to fall in between the model relations for energy injection of 0.5 keV and 0.75 keV per particle after correcting for the effects of cooling in groups. It should be noted that for galaxy groups, scatter in both dark matter concentration and energy injection may contribute to the scatter in the relation along with cooling flows and other factors.

It therefore appears that the mean trends in the mass, β and luminosity with system temperature can be explained reasonably well by a combination of variation in the dark matter halo concentration and energy injection into the ICM. The energy injection that best reproduces these trends is in the range $\sim 0.5 - 0.75$ keV per particle. This is comparable with the value of 0.44 keV per particle found by Lloyd-Davies et al. (2000) using analytical models fitted to X-ray spectral images of galaxy clusters and groups. Bower et al. (2001) derive a value of 0.6-1 keV per particle when comparing their model with observational data which is also comparable with our result. There is considerable scatter in

all the relations which may be due to a combination of scatter in the concentration parameter and energy injection.

In the case of the β-temperature relation it is possible to derive excess energies for individual systems and in general these are higher than the values derived for the relations as a whole. The lowest mass systems however appear to have systematically lower excess energies and it is possible that this is due to selection effects where low mass systems with high excess energies are not observed because they are not luminous enough. The mean excess energy of the systems below 3 keV excluding the three lowest mass systems is $\sim$1 keV per particle. This suggests the possibility that care must be taken when deriving mean excess energies from cluster relations, since the lowest mass systems which tend to have the largest influence on the result may have systematically lower excess energies due to selection effects. A value of ~ 0.5 keV per particle therefore appears to be a lower limit on the mean excess energy in galaxy clusters and groups, and the true mean value may be as high as 1 keV per particle.

It is possible to estimate the amount of energy that could be available as a result of star formation in order to assess whether it is possible for this process alone to account for the excess energy we observe. Assuming that each supernova produces 10^{51} erg of kinetic energy (Woosley & Weaver 1986), there is 0.007 supernova per $M_{\odot}$ of stars (Bower et al. 2001) and the gas and stellar fraction of galaxy clusters and groups are 0.2 and 0.11 respectively (Lloyd-Davies et al. 2000), results in an energy per particle of 1.2 keV. There are considerable uncertainties in the assumptions made in this derivation but it does suggest that it is possible for star formation alone to provide the excess energy we observe, although the higher mean excess energy of $\sim$1 keV per particle we derive for individual systems would require a very high efficiency in transferring the energy from supernovae to the intergalactic medium. Detailed observation of the metal enrichment of the ICM (Finoguenov et al. 2000) also support a scenario of heating of the ICM prior to cluster formation by galaxy winds.

It seems clear that the approach of modeling similarity-breaking in galaxy clusters and comparing the predictions to observations is extremely useful in constraining the processes involved in the formation and evolution of galaxy clusters. With the advent of observation of large samples of galaxy clusters using XMM-Newton and Chandra it should be possible to construct much more detailed models which place much greater constraints on the structure of galaxy clusters. It may also be possible to directly test the temperature profiles predicted by our models with such observations.

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